18
Stability of Slopes

18.1. INTRODUCTION

An earth slope is an unsupported, inclined surface of a soil mass. Earth slopes are formed for railway formations, highway embankments, earth dams, canal banks, levees, and at many other locations. Fig 18.1 shows some of the examples of earth slopes.

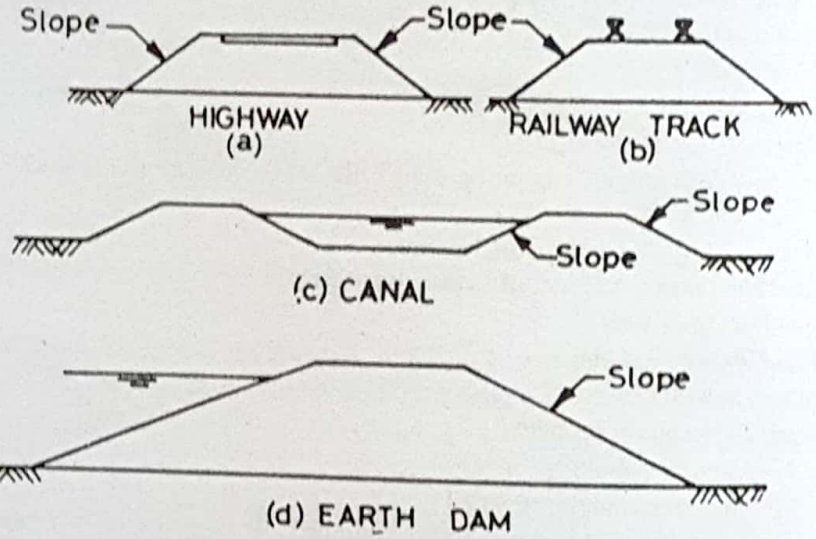

Fig. 18.1. Earth Slopes.

The cost of earth work would be minimum if the slopes are made steepest. However, very steep slopes may not be stable. A compromise has to be made between economy and safety, and the slopes provided are neither too steep nor too flat. In other words, the steepest slopes which are stable and safe should be provided.

The failure of a slope may lead to loss of life and property. It is, therefore, essential to check the stability of proposed slopes. With the development of modern methods of testing of soils and stability analysis, a safe and economical design of a slope is possible. The geotechnical engineer should have a thorough knowledge of the various methods for checking the stability of slopes and their limitations.

The failure of a soil mass occurs along a plane or a curved surface when a large mass of soil slides with respect to the remaining mass. In general, there is a downward and outward movement of the soil mass. A slope failure occurs when the forces causing failure are greater than the shearing resistance (shear strength) developed along a critical surface of failure. The factors leading to the failure of slopes may be classified into two categories.

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(1) *The factors which cause an increase in the shear stresses.* The stresses may increase due to weight of water causing saturation of soils, surcharge loads, seepage pressure or any other cause. The stresses are also increased due to steepening of slopes either by excavation or by natural erosion.

(2) *The factors which cause a decrease in the shear strength of the soil.* The loss of shear strength may occur due to an increase in water content, increase in pore water pressure, shock or cyclic loads, weathering or any other cause.

Most of the natural slope failures occur during rainy seasons, as the presence of water causes both increased stresses and the loss of strength.

18.2. BASIS OF ANALYSIS

The soil mass must be safe against slope failure on any conceivable surface across the slope. Although the methods using the theory of elasticity or plasticity are also being increasingly used, the most common methods are based on limiting equilibrium in which it is assumed that the soil is at the verge of failure. The methods of limiting equilibrium are statically indeterminate. As the stress-strain relationships along the assumed surface are not known, it is necessary to make assumptions so that the system becomes statically determinate and it can be analysed easily using the equations of equilibrium. The following assumptions are generally made.

(1) The stress system is assumed to be two-dimensional. The stresses in the third direction (perpendicular to the section of the soil mass) are taken as zero.

(2) It is assumed that the Coulomb equation for shear strength is applicable and the strength parameters c and ϕ are known.

(3) It is further assumed that the seepage conditions and water levels are known, and the corresponding pore water pressure can be estimated.

(4) The conditions of plastic failure are assumed to be satisfied along the critical surface. In other words, the shearing strains at all points of the critical surface are large enough to mobilise all the available shear strength.

(5) Depending upon the method of analysis, some additional assumptions are made regarding the magnitude and distribution of forces along various planes.

In the analysis, the resultant of all the actuating forces trying to cause the failure is determined. An estimate is also made of the available shear strength. The factor of safety of the slope is determined from the available resisting forces and the actuating forces.

18.3. DIFFERENT DEFINITIONS OF FACTORS OF SAFETY

Three different definitions of the factor of safety are used.

(a) Factor of safety with respect to shear strength

In common usage, the factor of safety is defined as the ratio of the shear strength to the shear stress along the surface of failure. The factor of safety as defined above is known as the factor of safety with respect to shear strength.

Thus
$$F_s = \frac{s}{\tau_m} \qquad \ldots(18.1)$$

where F_s = factor of safety with respect to shear strength, s = shear strength,

τ_m = mobilised shear strength (equal to applied shear stress).

Eq. 18.1 can be written in terms of the cohesion intercept and the angle of shear resistance as

$$F_s = \frac{c + \bar{\sigma} \tan \phi}{c_m + \bar{\sigma} \tan \phi_m} \qquad \ldots(18.2)$$

where c_m = mobilised cohesion, ϕ_m = mobilised angle of shear resistance, $\bar{\sigma}$ = effective pressure.

Rearranging Eq. 18.2,

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$$\frac{c}{F_s} + \frac{\bar{\sigma} \tan \phi}{F_s} = c_m + \bar{\sigma} \tan \phi_m$$

Therefore, $c_m = c/F_s$ ...(18.3)

and $\tan \phi_m = \tan \phi / F_s$ ...(18.4)

Eqs. 18.3 and 18.4 indicate that the factor of safety with respect to the cohesion intercept and that with respect to the angle of shearing resistance are equal to the factor of safety with respect to the shear strength.

(b) Factor of safety with respect to cohesion

The factor of safety with respect to cohesion (F_c) is the ratio of the available cohesion intercept (c) and the mobilised cohesion intercept.

Thus $$F_c = \frac{c}{c_m}$$ ...(18.5)

where c = cohesion intercept, c_m = mobilised cohesion intercept,

F_c = factor of safety with respect to cohesion.

(c) Factor of safety with respect to friction

The factor of safety with respect to friction is the ratio of the available frictional strength to the mobilised frictional strength. Thus

$$F_\phi = \frac{\bar{\sigma} \tan \phi}{\bar{\sigma} \tan \phi_m}$$

or $$F_\phi = \frac{\tan \phi}{\tan \phi_m}$$ ...(18.6)

where F_ϕ = factor of safety with respect to friction, ϕ = angle of shearing resistance,

ϕ_m = angle of mobilised shearing resistance.

For small angles, Eq. 18.6 can be expressed as

$$F_\phi = \frac{\phi}{\phi_m}$$ ...([18.6(a)]

In the analysis of stability of slopes, generally the three factors of safety are taken equal, *i.e.* $F_s = F_c = F_\phi$. However, sometimes when greater reliance is placed on the parameter ϕ than the parameter c, the factor of safety with respect to cohesion is taken greater than that with respect to friction. In such a case, the factor of safety with respect to friction is usually taken as unity *i.e.* $\phi_m = \phi$.

18.4. TYPES OF SLOPE FAILURES

A slope may have any one of the following types of failures.

(1) **Rotational failure.** This type of failure occurs by rotation along a slip surface by downward and outward movement of the soil mass (Fig. 18.2). The slip surface is generally circular for homogeneous soil conditions and non-circular in case of non- homogeneous conditions. Rotational slips are further divided into 3 types.

(*a*) Toe failure, in which the failure occurs along the surface that passes through the toe [Fig. 18.2 (*a*)].

(*b*) Slope failure, in which the failure occurs along a surface that intersects the slope above the toe [Fig. 18.2 (*b*)].

(*c*) Base failure, in which the failure surface passes below the toe [Fig. 18.2 (*c*)].

The slope failure occurs when a weak plane exists above the toe. The base failure occurs when a weak stratum lies beneath the toe. If a strong stratum exists below the toe, the slip surface of the base failure is tangential to that stratum. In all other cases, the failures are generally toe failures. Toe failures are most common.

(2) **Translational Failure.** A constant slope of unlimited extent and having uniform soil properties at the

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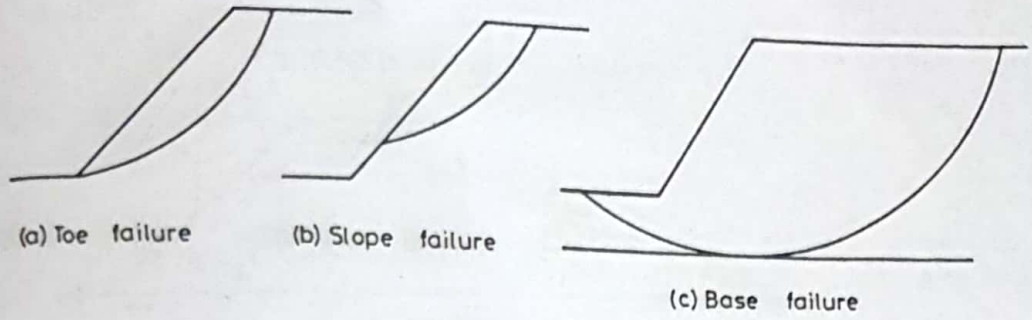

Fig. 18.2. Rotational Failure.

same depth below the free surface is known as an infinite slope. In practice, the slopes which are of considerable extent and in which the conditions on all verticals are adequately represented by average conditions are designated as *infinite slopes*.

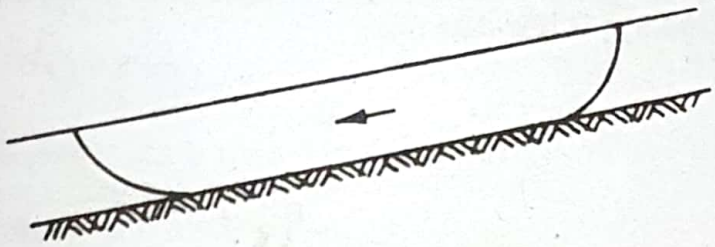

Fig. 18.3. Translational Failure.

Translational failure occurs in an infinite slope along a long failure surface parallel to the slope (Fig. 18.3). The shape of the failure surface is influenced by the presence of any hard stratum at a shallow depth below the slope surface. Translational failures may also occur along slopes of layered materials.

(3) **Compound Failure.** A compound failure is a combination of the rotational slips and the translational slip (Fig. 18.4). A compound failure surface is curved at the two ends and plane in the middle portion. A compound failure generally occurs when a hard stratum exists at considerable depth below the toe.

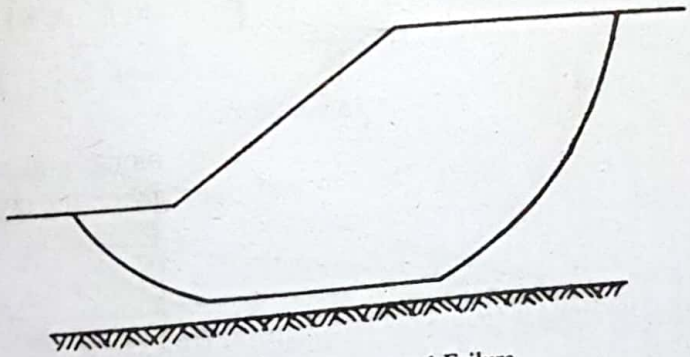

Fig. 18.4. Compound Failure.

(4) **Wedge Failure.** A failure along an inclined plane is known as plane failure or wedge failure or block failure (Fig. 18.5). It occurs when distinct blocks and wedges of the soil mass become separated.

A plane failure is similar to translational failure is many respects. However, unlike translational failure which occurs in an infinite slope, a plane failure may occur even in a finite slope consisting of two different materials or in a homogeneous slope having cracks, fissures, joints or any other specific plane of weakness.

(5) **Miscellaneous Failures.** In addition to above four types of failures, some complex types of failures

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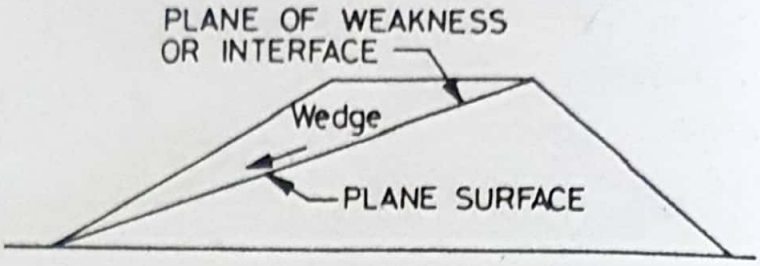

Fig. 18.5. Wedge Failure.

in the form of spreads and flows may also occurs.

18.5. STABILITY OF AN INFINITE SLOPE OF COHESIONLESS SOILS

The stability criteria of an infinite slope of cohesionless soils will depend whether the soil is dry, or submerged or has steady seepage, as explained below.

(1) **Dry Soil.** Fig. 18.6 (*a*) shows a section of an infinite slope having a slope angle of *i*. Let us consider the prism *ABCD* of the soil, with the inclined length *AB* equal to *b*. The horizontal length of the prism is *b* cos *i*. The height of the prism is *H* [Fig. 18.6 (*b*)].

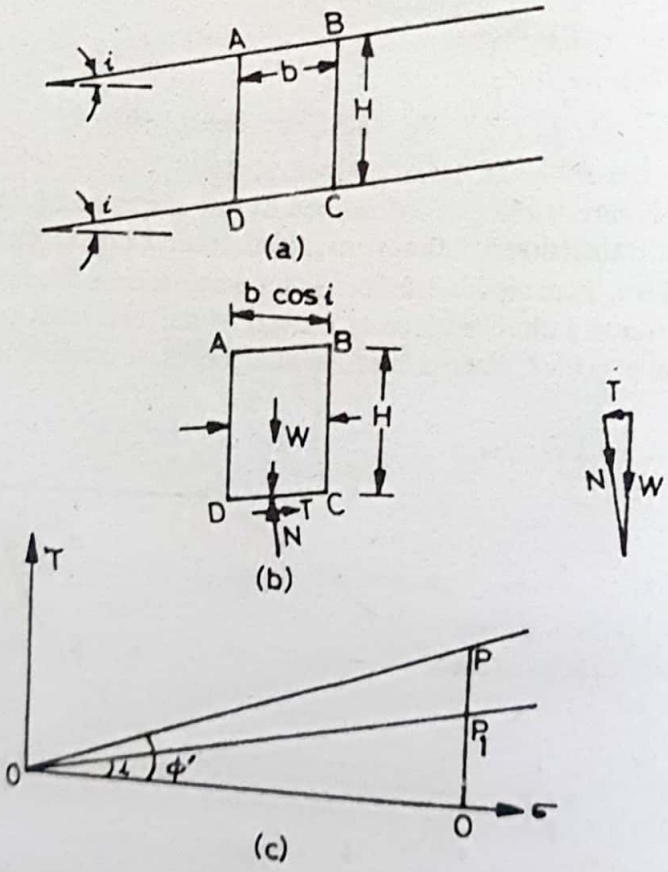

Fig. 18.6. Infinite Slope in Dry Sand.

Volume of prism per unit length = $Hb \cos i$

Weight of prism per unit length, $W = \gamma (Hb \cos i)$

The weight of the prism can be resolved into the normal component N and tangential components T to plane *CD*.

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Thus
$$N = W\cos i = \gamma\, Hb\, \cos^2 i$$
$$T = W\sin i = \gamma\, Hb \cos i \sin i$$

The normal and shear stresses are given by

$$\sigma = \frac{N}{b} = \frac{\gamma\, Hb\cos^2 i}{b} = \gamma H\cos^2 i \qquad \ldots(18.7)$$

$$\tau = \frac{T}{b} = \frac{\gamma\, Hb\cos i\sin i}{b} = \gamma H\cos i\sin i \qquad \ldots(18.8)$$

The shear stresses τ tend to cause the shear failure along CD. This tendency is opposed by the shearing resistance developed along the plane CD. As the soil is dry, there is no pore water pressure.

Therefore,
$$s = \bar{\sigma}\tan\phi' = \sigma\tan\phi'$$

or
$$s = (\gamma H\cos^2 i)\tan\phi'$$

The factor of safety against shear failure is given by

$$F_s = \frac{s}{\tau} = \frac{(\gamma H\cos^2 i)\tan\phi'}{\gamma H\cos i\sin i}$$

or
$$F_s = \frac{\tan\phi'}{\tan i} \qquad \ldots(18.9)$$

Eq. 18.9 indicates that the slope is just stable when $\phi' = i$. The factor of safety is greater than unity when i is less than ϕ'. For the slope angle i greater than ϕ', the slope is not stable.

It is worth noting that the factor of safety of an infinite slope of a cohesionless soil is independent of the height H of the assumed failure prism.

The angle ϕ' in Eq. 18.9 should correspond to the actual relative density of the soil. As the soil in the surface layers is in a relatively loose state, the angle ϕ' corresponding to the loose state is generally taken.

Eq. 18.9 can be represented graphically [Fig. 18.6 (*c*)]. The ordinate PQ is equal to $\sigma\tan\phi'$ and represents the shear strength. The ordinate P_1Q represents the shear stress τ equal to $\sigma\tan i$. Obviously, the factor of safety is given by

$$F_s = \frac{s}{\tau} = \frac{PQ}{P_1Q} = \frac{\tan\phi'}{\tan i}$$

(2) **Submerged Slope.** If the slope is submerged under water, the normal effective stress and the shear stress are calculated using the submerged unit weight and not the bulk unit weight as was used for dry soil. Thus, from Eqs. 18.7 and 18.8.

$$\bar{\sigma} = \gamma' H\cos^2 i \qquad \ldots(18.10)$$

and
$$\tau = \gamma' H\sin i\cos i \qquad \ldots(18.11)$$

where γ' is the submerged unit weight.

Therefore, the factor of safety is given by

$$F_s = \frac{s}{\tau} = \frac{(\gamma' H\cos^2 i)\tan\phi'}{\gamma' H\sin i\cos i}$$

or
$$F_s = \frac{\tan\phi'}{\tan i} \qquad \ldots(18.12)$$

Comparing Eqs. 18.9 and 18.12, it is observed that the factor of safety of a submerged slope is the same as that in dry condition.

(3) **Steady Seepage along the slope.** Fig. 18.7 (*a*) shows an infinite slope with steady seepage parallel to the surface. Fig. 18.7 (*b*) shows a free-body diagram of the prism $ABCD$ of the soil. The forces acting on the vertical sides of the prism due to water and soil are equal and opposite and, therefore, cancel. The weight of the prism W is taken corresponding to the saturated conditions.

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Therefore, $W = \gamma_{sat} H b \cos i$

$$N = W \cos i = \gamma_{sat} H b \cos^2 i$$

$$T = W \sin i = \gamma_{sat} H b \sin i \cos i$$

At the base of the prism, there is an upward force due to water pressure (u), given by

$$u = \gamma_w H \cos^2 i$$

Uplift force, $U = (\gamma_w H \cos^2 i)\, b$

Thus, the net normal force $\bar{N}$ is given by

$$\bar{N} = N - U = \gamma_{sat} H b \cos^2 i - (\gamma_w H \cos^2 i)\, b$$

or $\bar{N} = \gamma' bH \cos^2 i$, where γ' is submerged unit weight.

The effective stress is given by

$$\bar{\sigma} = \frac{\bar{N}}{b} = \gamma' H \cos^2 i$$

Shear strength, $s = \bar{\sigma} \tan \phi' = \gamma' H \cos^2 i \tan \phi'$

The shear stress is given by

$$\tau = \frac{T}{b} = \gamma_{sat} H \sin i \cos i$$

Therefore, the factor of safety is given by

$$F_s = \frac{s}{\tau} = \frac{\gamma' H \cos^2 i \tan \phi'}{\gamma_{sat} H \sin i \cos i}$$

or
$$F_s = \frac{\gamma' \tan \phi'}{\gamma_{sat} \tan i} \quad \ldots(18.13)$$

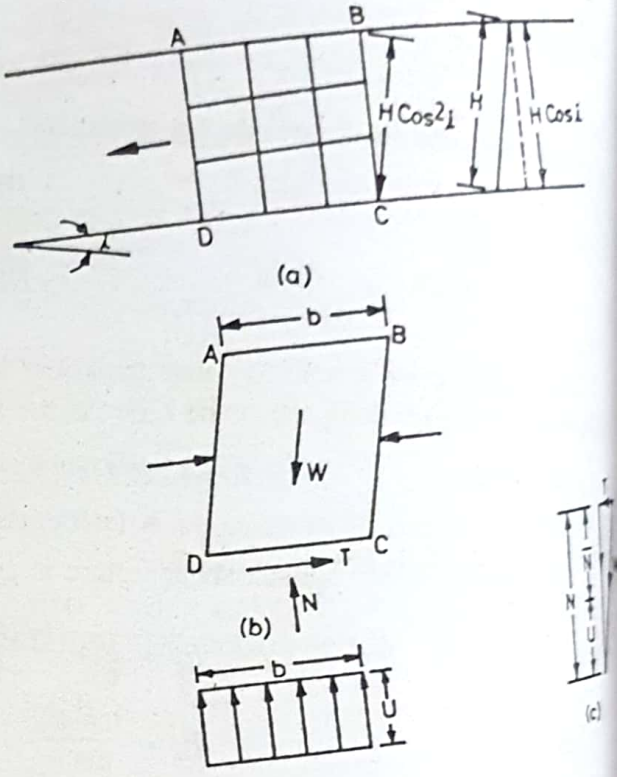

Fig. 18.7. Steady Seepage along the Slope.

As the submerged unit weight γ' is about one-half of the saturated unit weight, the factor of safety of the slope is reduced to about one-half of that corresponding to the condition when there is no seepage. The angle ϕ' in the wet condition of a cohesionless soil is approximately the same as in dry condition.

18.6. STABILITY ANALYSIS OF AN INFINITE SLOPE OF COHESIVE SOILS

The stability analysis of an infinite slope of cohesive soils is similar to that in the case of cohesionless soils, with one basic difference that the shear strength of a cohesive ($c - \phi$ soil) is given by

$$s = c' + \bar{\sigma} \tan \phi'$$

(a) Dry Soil. Taking values of $\bar{\sigma}$ and τ from Eqs. 18.7 and 18.8, we have

$$s = c' + (\gamma H \cos^2 i) \tan \phi'$$

$$\tau = \gamma H \cos i \sin i$$

Therefore, the factor of safety F_s is given by

$$F_s = \frac{c' + (\gamma H \cos^2 i) \tan \phi'}{\gamma H \cos i \cos i} \quad \ldots(18.14)$$

Thus, the factor of safety of an infinite slope in cohesive soils depends not only on ϕ' and i but also on γ, H and c'.

Fig. 18.8 represents a graphical method for the determination of the factor of safety. The line RP_2 represents the failure envelope. When the slope angle i is less than ϕ', the slope is always safe as shown by line OP_1. When the slope angle i_2 is greater than ϕ', the slope line cuts the failure envelope. At point P, the slope is just stable. For normal stress greater than that indicated by point P, the shear stress is greater than

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the shear strength and the slope is not stable.

As the normal stress σ depends upon the height H of the slope, an expression for the height can be found when the slope is just stable. Therefore, equating the shear stress and the shear strength corresponding to point P,

$$\gamma H \cos i \sin i = c' + \gamma H \cos^2 i \tan \varphi'$$

or $$\gamma H \cos^2 i \left(\frac{\sin i}{\cos i} - \tan \varphi' \right) = c'$$

or $$H = \frac{c'}{\gamma (\tan i - \tan \varphi') \cos^2 i}$$

The height at which the slope is just stable is known as the critical height (H_c). Thus

$$H_c = \frac{c'}{\gamma (\tan i - \tan \varphi') \cos^2 i} \qquad ...(18.15)$$

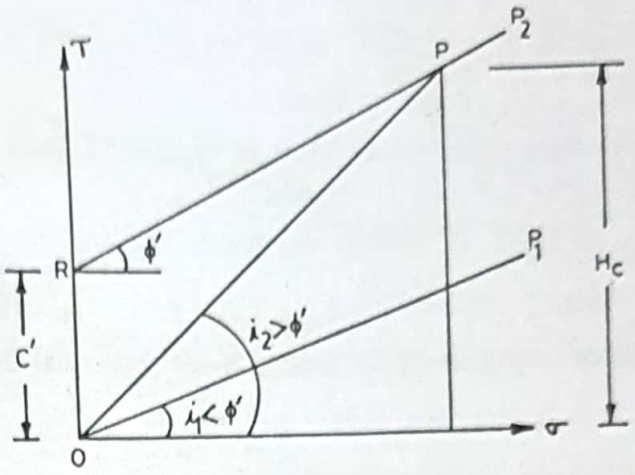

Fig. 18.8. Infinite Slope in Cohesive Soils.

For heights less than the critical height, the factor of safety is given by Eq. 18.14.

(b) Submerged Slope

As in the case of cohesionless soils, the normal and tangential components of the weight are taken for submerged unit weights and not for bulk unit weights. Thus, from Eq. 18.14,

$$F_s = \frac{c' + \gamma' H \cos^2 i \tan \varphi'}{\gamma' H \cos i \sin i} \qquad ...(18.16)$$

The value of ϕ' in Eq. 18.16 should be taken corresponding to the submerged conditions, which may be quite different from that in dry condition in case of cohesive soils.

(c) Steady Seepage along the Slope

The case is similar to that for a cohesionless soil. In this case, the factor of safety is given by

$$F_s = \frac{c' + \gamma' H \cos^2 i \tan \varphi'}{\gamma_{sat} H \cos i \sin i} \qquad ...(18.17)$$

The critical height is obtained corresponding to a factor of safety of unity. Thus

$$c' + \gamma' H_c \cos^2 i \tan \varphi' = \gamma_{sat} H_c \cos i \sin i$$

or $$H_c \cos^2 i (\gamma_{sat} \tan i - \gamma' \tan \varphi') = c'$$

or $$H_c = \frac{c'}{(\gamma_{sat} \tan i - \gamma' \tan \varphi') \cos^2 i} \qquad ...(18.18)$$

Sometimes, Eq. 18.18 is written as

$$H_c = \frac{c'}{\gamma_{sat} \left[\tan i - \left(\frac{\gamma'}{\gamma_{sat}} \right) \tan \phi' \right] \cos^2 i} \qquad ...(18.19)$$

Eq. 18.19 indicates that the effect of the angle of shearing resistance ϕ' is reduced as compared with Eq. 18.15 of the dry soil.

18.7. WEDGE FAILURE

A wedge failure occurs when a soil deposit has a specific plane of weakness. The stratified deposits generally fail along the interface. Fig. 18.9 shows a soil mass resting on an inclined layer of impermeable soil. There is a tendency of the upper mass to slide downward along the plane of contact AB.

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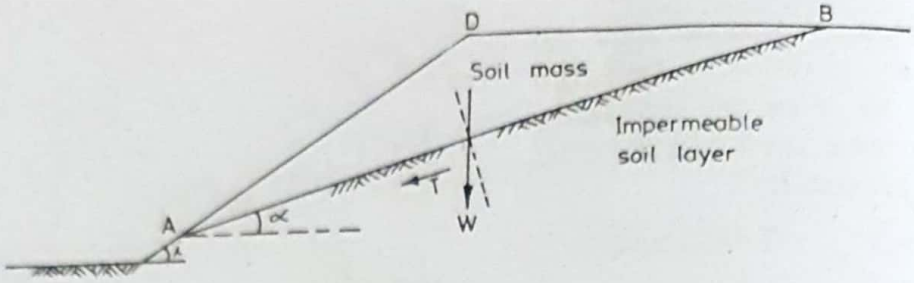

Fig. 18.9.

The force trying to cause sliding is the tangential component T of the weight (W) along the plane of contact.

$$T = W \sin \alpha \qquad \ldots(a)$$

where α is the angle which the plane AB makes with horizontal, W is the weight of wedge per unit length perpendicular to the plane of paper.

The force tending to resist the sliding depends upon the cohesion c and the frictional force and is given by

$$S = cL + (W \cos \alpha) \tan \phi \qquad \ldots(b)$$

where L is the length of the failure surface AB.

The factor of safety against sliding is obtained from (a) and (b) as

$$F_s = \frac{cL + (W \cos \alpha) \tan \varphi}{W \sin \alpha} \qquad \ldots(18.20)$$

18.8. CULMANN'S METHOD

Culmann's method is used for the approximate stability analysis of homogeneous slopes. A plane failure surface passing through the toe is assumed. A plane failure surface is not a correct assumption for a homogeneous soil. However, it is a simple failure mechanism and is described for purpose of illustration and for determination of the approximate value of the factor of safety.

Let us consider the equilibrium of the triangular wedge ABD formed by the assumed failure surface AB (Fig. 18.10) The wedge is in equilibrium under the three forces:

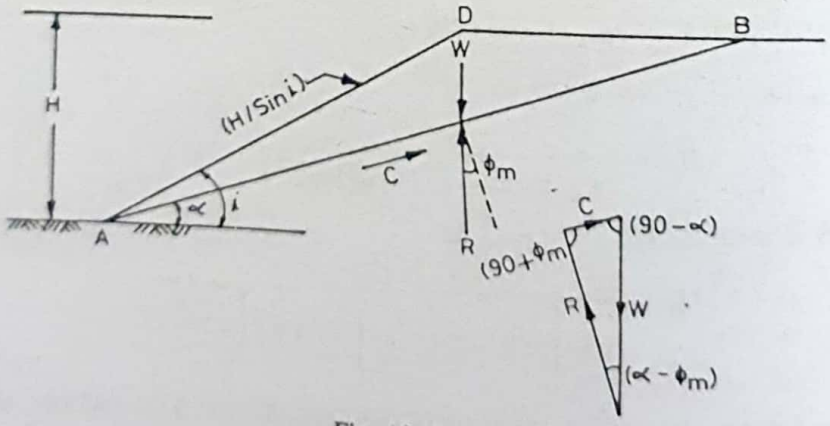

Fig. 18.10.

(1) Weight of the wedge (W).

(2) Cohesive force (C) along the surface AB.

(3) Reaction R. The reaction R is inclined at angle ϕ_m to the normal.

The triangle of forces is also shown in figure. The magnitude and direction of W and C are known. The direction of R is also known. The weight of the wedge is given by

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$$W = \frac{1}{2}\gamma L \left(\frac{H}{\sin i}\right) \sin (i - \alpha) \qquad \ldots(a)$$

and $$C = c_m L$$

where H = height of slope, c_m = mobilised cohesion, ϕ_m = angle of mobilised friction, and L = length of failure surface AB.

[Note. Area of a triangle $ABD = \frac{1}{2} d\, b \sin A$ where d and b are the length of sides BA and AD, respectively.]

From the law of sines, $$\frac{C}{W} = \frac{\sin (\alpha - \phi_m)}{\sin (\phi_m + 90°)} = \frac{\sin (\alpha - \phi_m)}{\cos \phi_m} \qquad \ldots(c)$$

Substituting the values of W and C from Eqs. (a) and (b) in Eq. (c),

$$\frac{c_m L}{1/2\, \gamma L\, (H/\sin i)\, \sin (i - \alpha)} = \frac{\sin (\alpha - \phi_m)}{\cos \phi_m} \qquad \ldots(d)$$

or $$\left(\frac{c_m}{\gamma H}\right) = \frac{1}{2} \operatorname{cosec} i \quad \sin (i - \alpha) \quad \sin (\alpha - \phi_m) \sec \phi_m \qquad \ldots(18.21)$$

The left-hand side of Eq. 18.21 is known as the *stability number* (S_n). The most dangerous plane is that for which the angle α is such that the stability number becomes a *maximum, i.e.*,

$$\frac{d(S_n)}{d\alpha} = 0$$

or $$\frac{d}{d\alpha} [\sin (i - \alpha) \sin (\alpha - \varphi_m)] = 0$$

or $$\sin (i - \alpha) \cos (\alpha - \phi_m) - \sin (\alpha - \phi_m) \cos (i - \alpha) = 0$$

or $$\tan (i - \alpha) = \tan (\alpha - \phi_m)$$

or $$i - \alpha = \alpha - \phi_m$$

or $$\alpha_c = \frac{(i + \phi_m)}{2} \qquad \ldots(e)$$

where α_c is the critical slope angle.

From Eqs. 18.21, taking the value of α_c from Eq. (e),

$$\left(\frac{c_m}{\gamma H}\right)_{max} = \frac{1}{2} \operatorname{cosec} i \sec \phi_m \left[\sin \left\{ i - \left(\frac{i + \phi_m}{2}\right)\right\}\right] \times \left[\sin \left\{\left(\frac{i + \phi_m}{2}\right) - \phi_m\right\}\right]$$

or $$= \frac{1}{2} \operatorname{cosec}\, i\, \sec\, \phi_m \sin \left(\frac{i - \phi_m}{2}\right) \sin \left(\frac{i - \phi_m}{2}\right)$$

or $$= \frac{1}{2} \operatorname{cosec}\, i\, \sec\, \phi_m \left[\frac{i - \cos\, (i - \phi_m)}{2}\right]$$

or $$\left(\frac{c_m}{\gamma H}\right)_{max} = \frac{1 - \cos\, (i - \phi_m)}{(4 \sin i \cos \phi_m)} \qquad \ldots(18.22)$$

or $$H = \frac{4\, c_m \sin i \cos \phi_m}{\gamma [1 - \cos (i - \phi_m)]} \qquad \ldots[18.22(a)]$$

where H is the safe height of slope.

The Culmann method gives reasonably accurate results for homogeneous slopes which are vertical or

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nearly vertical. The critical surface for general slopes is not a plane and, therefore, the critical slope α_c has little practical use for such slopes.

18.9. $\phi_u = 0$ ANALYSIS

In case of fully saturated clay under undrained conditions, the stability of the slope can be checked in terms of total stresses. Such a condition occurs in slopes immediately after construction. In this case, $\phi_u = 0$ and $c = c_u$.

The failure surface is assumed to be a circular arc AB (Fig. 18.11). The figure shows a failure surface with centre O and radius r (*i.e.* $OB = OA = r$).

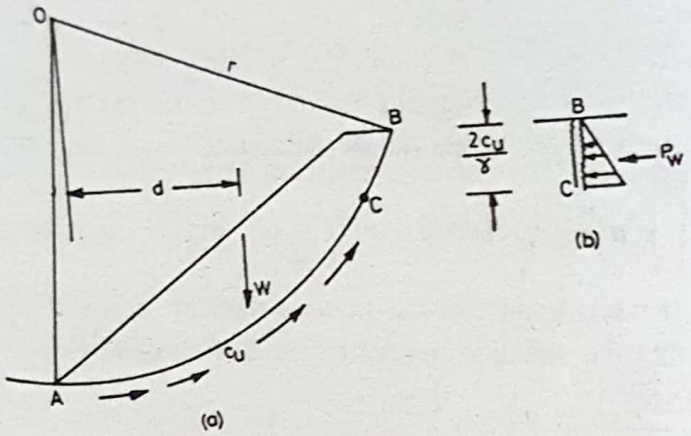

Fig. 18.11.

The total weight W above the failure surface causes instability. For equilibrium, the shear strength to be mobilised along the failure surface can be expressed as

$$\tau_m = \frac{s}{F} = \frac{c_u}{F}$$

where F is the factor of safety.

Taking moments about O,

$$W \times d = (c_u/F) L_a r \qquad \ldots(18.23)$$

where L_a is the length of arc AB and d is the lever arm of W about O.

Thus

$$F = \frac{c_u L_a r}{W \times d} \qquad \ldots[18.23(a)]$$

If a tension crack develops and water enters the crack, the hydrostatic pressure force P_w acts on the portion BC of the arc at a height of $h/3$ from C, where h is the depth of tension crack, equal to $2\,c_u/\gamma$ (see chapter 19). The arc length in that case should be taken equal to AC. Eq. 18.23 may be modified accordingly.

18.10. FRICTION CIRCLE METHOD

The friction circle method is useful for the stability analysis of slopes made of homogeneous soils. In this method, the slip surface is assumed to be an arc of a circle.

(a) Dry Soil

Fig. 18.11 shows a circular failure surface ABC of radius r with its centre at O. The three forces acting on the sliding wedge $AEBD$ are given below:

(1) Weight (W) of the sliding wedge.

(2) Cohesive force C developed along the slip surface AEB.

(3) Reaction R on the slip surface.

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The reaction R is inclined at angle ϕ_m to the normal to the slip surface. As the direction of the normal changes, the direction of R also changes. With the centre as O, a small circle, known as *friction circle*, is drawn with a radius $r \sin \phi_m$. All lines which are tangent to the friction circle make an angle ϕ_m with the normal of the slip surface. These lines represent the direction of the combined normal and mobilised frictional forces on the slip surface. The value of ϕ_m is obtained from Eq. 18.6, after choosing a value of F_ϕ. Thus the reaction R is tangential to the friction circle.

[**Note.** Actually, the reaction R is tangential to the friction circle of a slightly larger radius of $K\ r \sin \phi_m$ where K is a factor with a value greater than unity, as it is evident that the two reactions d_R [Fig. 18.12 (*a*)] intersect slightly outside the friction circle of radius $r \sin \varphi_m$. However, this discrepancy is generally disregarded].

The cohesive force C_m is equal to $c_m L_a$ where c_m is the mobilised cohesion and L_a is the length of the circular surface arc. It is convenient to replace this force acting along the arc by an equivalent force C acting along a line. The force along arc AEB is also equal in magnitude to the force $c_m \times L_c$ where L_c is the length of the chord AB. The line of action of this force can be determined by taking moments of the actual force and the equivalent force about O.

$$(L_c \times c_m) \times a = (c_m \times L_a) \times r$$

or

$$a = r \frac{L_a}{L_c} \qquad \ldots(18.24)$$

Obviously, the distance a is greater than r, as $L_a > L_c$.

The intersection of the weight W and the cohesive force C_m establishes a point P through which the

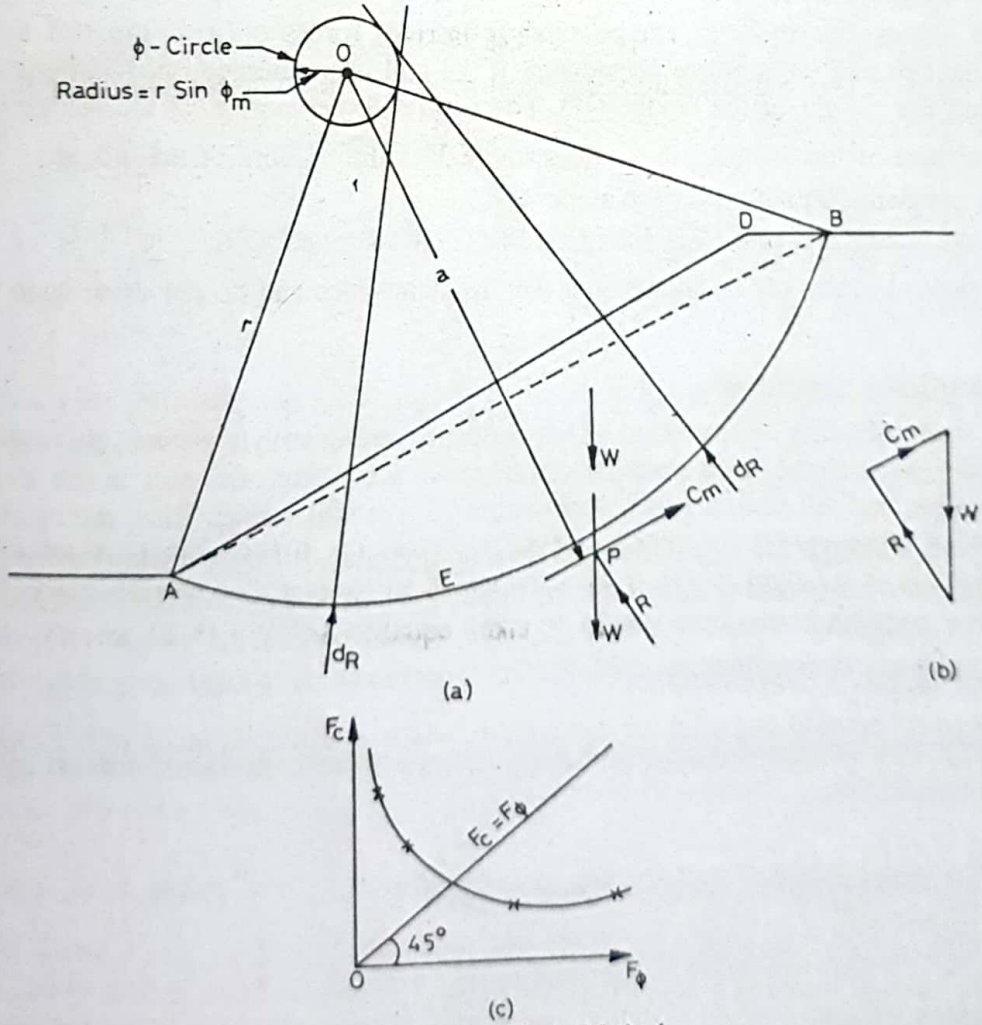

Fig. 18.12. Friction Circle Method.

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reaction R must act. The direction of R is obtained by drawing a line tangential to the ϕ-circle. The forces C_m and R can be determined from the force triangle.

Fig. 18.12 (*b*) shows the force triangle. The weight vector W is drawn first. The triangle is completed by drawing the vectors R and C_m along the directions already established. From the force triangle, the value of the cohesive force C_m is determined. The mobilised cohesion is equal to the cohesive force C_m divided by the length of the chord L_c. Thus

$$c_m = \frac{C_m}{L_c} \qquad \ldots(18.25)$$

The factor of safety with respect to cohesion is given by Eq. 18.5.

If the value of F_c obtained from Eq. 18.5 is not equal to the assumed value of F_ϕ, the analysis is repeated. The procedure is repeated after taking another trial surface. The slip circle which gives the minimum factor of safety (F_s) is the most critical circle. Generally, the analysis is repeated 3-4 times to obtain a curve between the assumed value of F_ϕ and the computed value of F_c, as shown in fig. 18.12 (*c*). The factor of safety with respect to shear strength F_s is obtained by drawing a line at 45°, which gives $F_c = F_\phi = F_s$.

For a purely cohesive soil, $\phi = 0$ and the friction circle reduces to a point. The factor of safety is determined from the resisting moment due to C and actuating moment due to W (See Sect. 18.9).

Sometimes, the factor of safety with respect to friction (F_ϕ) is assumed to be unity and the factor of safety with respect to only cohesion is obtained.

(*b*) Submerged slope

The above discussion refers to intergranular (effective) forces only. If the soil is dry and there is no submergence, the dry unit weights are considered. If the soil is submerged, the submerged unit weight is used when calculating the weight of the wedge (W). The neutral forces acting on the wedge are given below:

(1) Neutral part of the weight, $W_u = V\gamma_w$, where V is the volume of the wedge.
(2) Water pressure (U) acting on the slope AD.
(3) Resultant water pressure (U_B) acting on the curved surface AEB.

The three neutral forces are in equilibrium among themselves and do not draw upon the shear strength of the soil.

(*c*) Sudden drawdown conditions

When the water standing on the slope AD is suddenly and quickly removed, the water pressure force (U) disappears. However, if there is no time for drainage to occur from the soil in the slope, the soil remains submerged as before and the neutral part of the weight (W_u) is still acting. Thus, the equilibrium of the neutral forces is disturbed, although the equilibrium of the intergranular forces remains unaffected.

The equilibrium of the neutral forces can be maintained only if the soil can mobilise additional cohesion (c_a) such that the additional cohesive force $c_a L_a$, the neutral part of the weight (W_u) and the new resultant water pressure (U_B) are in equilibrium. The additional cohesion can be determined by drawing another force triangle.

The total cohesion mobilised (c_m') in this case is equal to the cohesion mobilised for intergranular forces and that for neutral forces.

$$c_m' = c_m + c_a \qquad \ldots(18.26)$$

The factor of safety with respect to cohesion is given by

$$F_c = \frac{c}{c_m'} = \frac{c}{c_m + c_a} \qquad \ldots(18.27)$$

Thus the factor of safety of the slope is considerably reduced during sudden drawdown conditions.

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18.11. STABILITY CHARTS

The stability number (S_n), as defined in Sect. 18.8, is given by

$$S_n = \frac{c_m}{\gamma H} = \frac{c}{F_c \gamma H} \qquad \ldots(18.28)$$

The reciprocal of the stability number is known as *stability factor.* The stability number is a dimensionless quantity.

Taylor determined the values of S_n for finite slopes using the friction circle method. Slopes that are of simple sections and of homogeneous soils may be analysed using the slope stability charts given by Taylor. The charts are prepared indicating the stability number, and slope angle i for various values of φ_m (Fig. 18.13). There are 5 parameter, *viz* c_m, γ, H, i and φ_m. However, if $\phi_m = 0$ (purely cohesive soils), a sixth

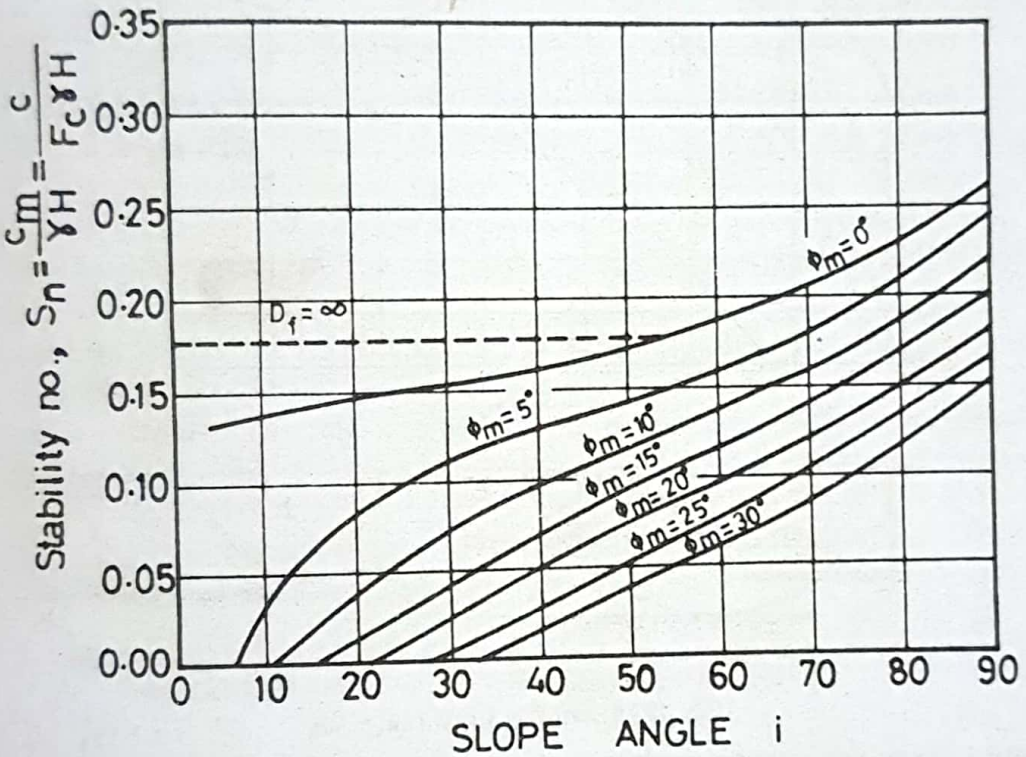

Fig. 18.13.

parameter D_f becomes also important (Fig. 18.14). The parameter D_f depends upon the depth of the hard stratum below the top of the slope, and is given by

$$D_f = \frac{\text{Depth of hard stratum below the top of the slope}}{\text{Height of slope}}$$

When the slope is steep, the failure surface passes through the toe, whereas for the flatter slope, the failure extends below the toe. The chart in Fig. 18.13 is based on the most critical circle passing through the toe of the slope.

For slope angle i greater then 53°, the toe failure occurs. For $i \leq 53°$, and small values of φ_m, a more critical surface may pass below the toe.

The chart in Fig. 18.14 is applicable for $\varphi_m = 0$. In soils with $\varphi_m = 0$ and the slope angle less than 53°, the failure surface extends below the toe as deep as possible. The stability number also depends upon the parameter D_f.

Uses

(1) The stability number can be used to determine the factor of safety of a given slope. For the known values of i and φ_m, the value of stability number (S_n) is determined from the chart in Fig. 18.13 (or Table 18.1) and the factor of safety is determined as

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$$F_c = \frac{c}{c_m} = \frac{c}{S_n \gamma H} \quad \ldots(18.25)$$

If $\varphi_m = 0$, the chart in Fig. 18.14 (or Table 18.2) is used to determine the stability no. (S_n) for the given values of i and D_f. The chart can also be used to determine the distance nH from the toe where the slip circle cuts the horizontal line.

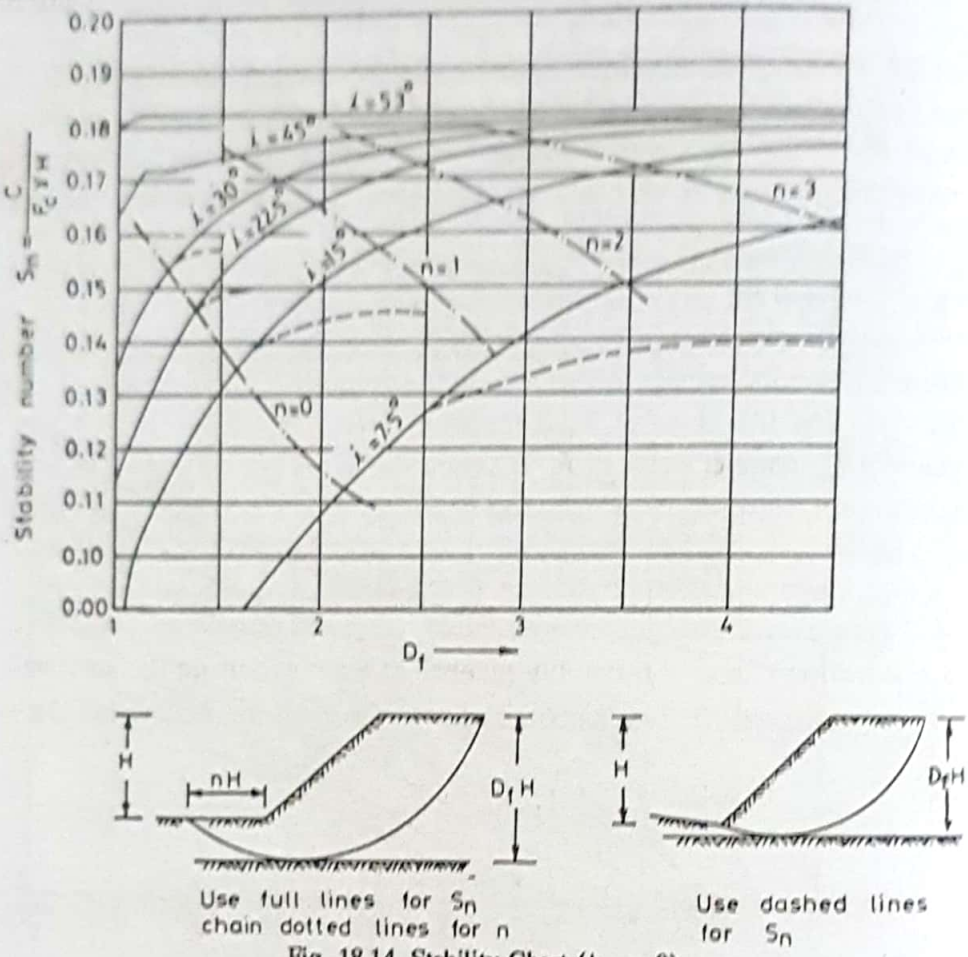

Fig. 18.14. Stability Chart ($\phi_m = 0$).

(2) The stability charts can also be used to determine the steepest slope for a given factor of safety. In this case, the stability number is computed from the relation

$$S_n = \frac{c}{F \gamma H}$$

For the computed value of S_n, the value of i is read from the stability chart for the given value of ϕ_m. Tables 18.1 and 18.2 give the values of stability numbers.

Table 18.1. Stability Numbers (see Fig. 18.13)

i \ ϕ_m	0°	5°	10°	15°	20°	25°
90°	0.261	0.239	0.218	0.199	0.182	0.166
75°	0.219	0.195	0.173	0.152	0.134	0.117
60°	0.191	0.162	0.138	0.116	0.097	0.079
45°	(0.170)	0.136	0.108	0.083	0.062	0.044
30°	(0.156)	(0.110)	0.075	0046	0.025	0.009
15°	(0.145)	(0.068)	0.070	(0.023)	—	—

[**Note.** Figures in brackets are for the most dangerous circles through the toe when a more dangerous circle exists below the toe].

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Table 18.2. Stability Numbers for Cohesive soils ($\phi_m = 0$) and $i \leq 53°$ (see Fig. 18.14)

D_f / i	1.0	1.50	2.0	3.0	∞
53°	0.181	0.181	0.181	0.181	0.181
45°	0.164	0.174	0.177	0.180	0.181
30°	0.133	0.164	0.172	0.178	0.181
22.5°	0.113	0.153	0.166	0.175	0.181
15°	0.083	0.128	0.150	0.167	0.181
7.5°	0.054	0.080	0.107	0.140	0.181

The following points should be carefully noted.

(1) If the factor of safety with respect to friction F_φ is unity, $\phi_m = \phi$.

(2) If the factor of safety with respect to shear strength is required, a trial and error procedure is adopted. A value of F_ϕ is assumed, and the value of ϕ_m is used to determine F_c from the stability chart. If F_c is not equal to assumed value of F_ϕ, another value of F_φ is assumed and the procedure is repeated. At least 3-4 trials are required to obtain a curve between F_c and F_ϕ, and to get the correct value of F_c from the curve, as shown in Fig. 18.12 (*c*).

(3) For a submerged slope, the stability number is computed using the submerged unit weight (γ'). The angle of shearing resistance should also be for the submerged conditions.

(4) For a sudden drawdown case, the stability number is computed using the saturated unit weight (γ_{sat}). The weighted angle of internal friction as obtained below is used for finding out the stability number.

$$\tan \phi_w = \frac{\gamma'}{\gamma_{sat}} \left(\frac{1}{F_\phi} \tan \varphi' \right) \qquad \ldots(18.30)$$

or

$$\phi_w = \frac{\gamma'}{\gamma_{sat}} \phi_m = \frac{\gamma'}{\gamma_{sat}} \left(\frac{\phi'}{F_\phi} \right) \qquad \ldots[18.30(a)]$$

where ϕ' is the effective angle of internal friction.

(5) For purely frictional soils, the cohesion intercept (c) is zero. As the stability number reduces to zero, the stability charts cannot be used for such soils.

(6) The values of shear strength parameter (c and ϕ) should be obtained from the tests conducted in the laboratory simulating the drainage conditions in the field. For example, for checking the stability of a slope of cohesive soils just after the constructions, the relevant drainage conditions are unconsolidated undrained conditions For long term stability, consolidated drained conditions are relevant.

18.12. SWEDISH CIRCLE METHOD

The actual shape of a slip surface in the case of finite slopes is curvilinear. For convenience, it is approximated as circular. The assumption of a circular slip surface and its application for stability analysis of slopes was developed in Sweden. The method is known as the Swedish circle method or the method of slices.

Fig. 18.15 (*a*) shows a slope. Let *AB* be a circular surface with radius r and centre *O*. The trial failure wedge above the slip surface is divided into vertical slices by drawing vertical lines, as shown. The slices are usually of equal width, but not necessarily so. In case of non-homogeneous slopes where the slip surface passes through more than one type of material, a vertical line is always located at the point where the slip surface passes from one material to the other.

Let us consider the equilibrium of one slice (say, No. 4). The slice is in equilibrium under the following forces.

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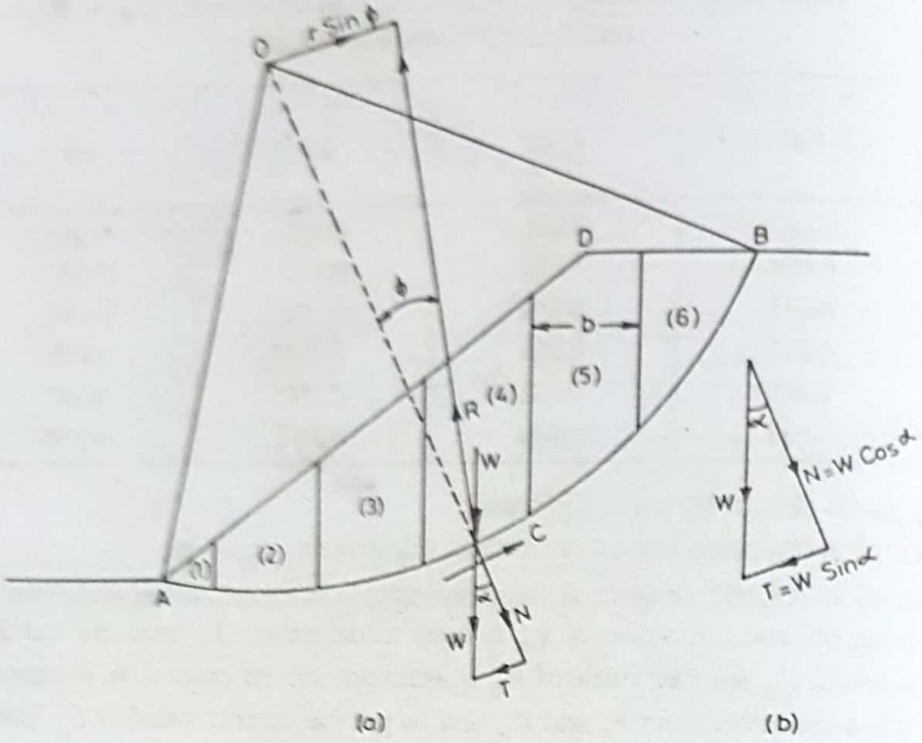

Fig. 18.15. Swedish Circle Methods.

(1) Weight (W) acting vertically through its centre of gravity.

(2) Cohesive force (C) acting along the curved surface in the direction opposite to the direction of probable movement of the wedge.

(3) Reaction (R) at the base inclined at angle ϕ to the normal, assuming the slippage is imminent.

(4) Reactions on the two vertical sides of the slice due to adjacent slices. However, in the Swedish circle method, it is assumed that the reactions on the two sides are equal and opposite and are, therefore, in equilibrium and do not affect the stability of the slice. Accordingly, only the first three forces are considered for the analysis.

The weight W is resolved into its normal component (N) and tangential component (T). Let us take the moments about the centre of rotation O of all the 3 forces.

Actuating or overturning moment, $M_0 = T \times r$ ...(a)

The moment due to N—components is zero, as those components always pass through O.

Resisting moment, $M_R = (C \Delta L) \times r + R (r \sin \phi)$ ...(b)

where ΔL is the length of the curved surface of the slice.

Resolving the forces is radial direction,

$$N = R \cos \phi \quad \text{or} \quad R = N / \cos \phi \qquad ...(c)$$

or $$R \sin \phi = N \tan \phi \qquad ...(d)$$

From Eqs. (*b*) and (*d*), $$M_R = (C \Delta L) r + N r \tan \phi \qquad ...(e)$$

The factor of safety for the slice is equal to the ratio of the resisting moment (M_R) and the overturning moment (M_0). Thus

$$F_s = \frac{r [c \Delta L + N \tan \phi]}{T r} = \frac{c \Delta L + N \tan \phi}{T}$$

The factor of safety of the entire wedge is given by

$$F_s = \frac{\Sigma c \Delta L + \Sigma N \tan \phi}{\Sigma T} \qquad ...(18.31)$$

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If c and ϕ are constant, $$F_s = \frac{c L_a + \tan \phi \Sigma N}{\Sigma T} \quad \ldots(18.32)$$

where L_a = length of the entire slip surface = $\Sigma \Delta L$

The components N and T are determined by drawing force triangles as shown in Fig. 18.15 (*b*). If the angle α which the normal makes with the vertical is measured, the components can be computed as under.

$$N = W \cos \alpha, \quad \text{and} \quad T = W \sin \alpha$$

The length ΔL of the arc is given by $b \sec \alpha$, where b is the width of the slice.

Therefore, Eq. 18.31 can be written as

$$F_s = \frac{\Sigma c b \sec \alpha + \Sigma (W \cos \alpha) \tan \phi}{\Sigma W \sin \alpha} \quad \ldots(18.33)$$

It may be noted that the tangential component T may be negative *i.e.* in the direction opposite to that of movement for some of the slices near the toe.

The procedure can be summarised as under:

(1) Take a trial wedge and divide it into 6 to 12 vertical slices.

(2) Determine the weight of each slice and its line of action.

For convenience, the weight is generally taken proportional to the middle ordinate of the slice and it is assumed to have line of its action through the middle of the slice.

(3) The weight is resolved (analytically or graphically) into normal and tangential components.

(4) The curved length ΔL of each slice is measured or computed.

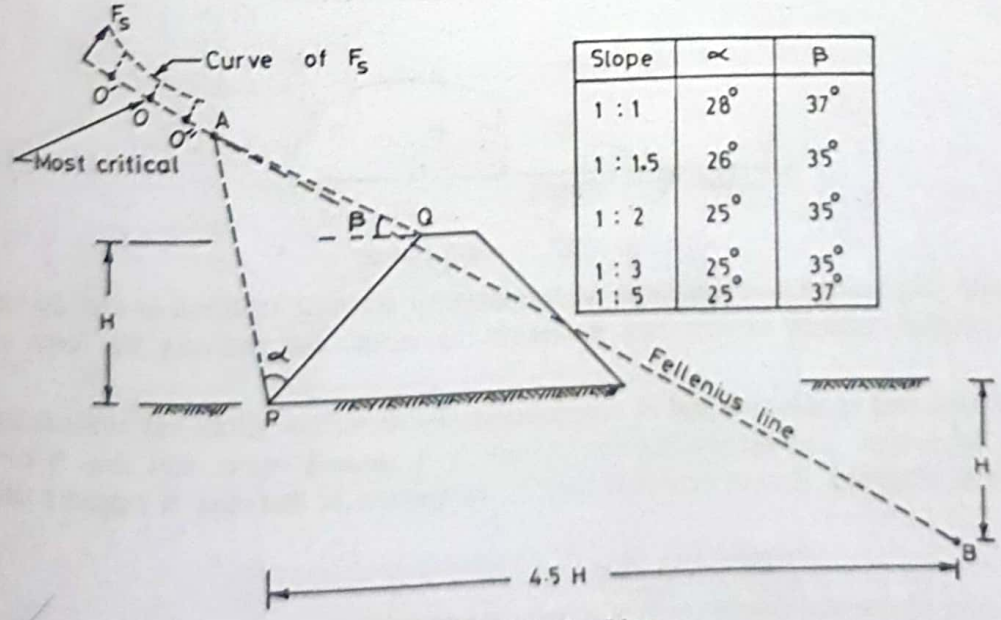
(5) The factor of safety is determined from Eq. 18.33 or Eq. 18.31.

The calculations are generally done in a tabular form. The stability analysis is repeated for a number of trial surfaces. The circle which gives the *minimum* factor of safety is the most critical circle.

Location of Most Critical Circle

In order to reduce the number of trials to locate the most critical circle, the Fellenius line AB can be drawn (Fig. 18.16). Fellenius has shown that the centre of the most critical circle lies on this line. For drawing the Fellenius line AB, the point B is located at a depth H and at a distance 4.5 H from point P at the toe of the slope, where H is the height of the slope. The point A is located by drawing two lines PA and QA, where PA makes angle α with the slope line PQ and QA makes angle β with the horizontal at Q. The angles α and β are obtained from the table given in Fig. 18.16. The angles depend upon the slope.

The centre of the most critical circle may lie anywhere on the line AB or its extension. The centres of



Slope	∝	β
1 : 1	28°	37°
1 : 1.5	26°	35°
1 : 2	25°	35°
1 : 3	25°	35°
1 : 5	25°	37°

Fig. 18.16. Fellenius Line.

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circle method of analysis discussed earlier. However, the charts are for a general case. These can be used for the evaluation of long-term stability analysis in terms of effective stresses. The charts can be used even in case of tension cracks.

(5) Stability charts for the analysis of earth dams under sudden drawdown conditions have been developed by Morgenstern (1963). The charts are based upon an effective stress analysis using the method of slices.

18.18. IMPROVING STABILITY OF SLOPES

The slopes which are susceptible to failure by sliding can be improved and made usable and safe. Various methods are used to stabilise the slopes. The methods generally involve one or more of the following measures, which either reduce the mass which may cause sliding or improve the shear strength of the soil in the failure zone.

(1) Slope flattening reduces the weight of the mass tending to slide. It can be used wherever possible.

(2) Providing a berm below the toe of the slope increases the resistance to movement. It is specially useful when there is a possibility of a base failure.

(3) Drainage helps in reducing the seepage forces and hence increases the stability. The zone of subsurface water is lowered and infiltration of the surface water is prevented.

(4) Densification by use of explosives, vibroflotation, or terra probe helps in increasing the shear strength of cohesionless soils and thus increasing the stability.

(5) Consolidation by surcharging, electro-osmosis or other methods helps in increasing the stability of slopes in cohesive soils.

(6) Grouting and injection of cement or other compounds into specific zones help in increasing the stability of slopes.

(7) Sheet piles and retaining walls can be installed to provide lateral support and to increase the stability. However, the method is quite expensive.

(8) Stabilisation of the soil helps in increasing the stability of slopes.

In the interest of economy, relatively inexpensive methods, such as slope flattening and drainage control, are generally preferred.

ILLUSTRATIVE EXAMPLES

Illustrative Example 18.1. *A long natural slope in an overconsolidated clay ($c' = 10 kN/m^2$, $\phi' = 25°$, $\gamma_{sat} = 20\ kN/m^2$) is inclined at 10° to the horizontal. The water table is at the surface and the seepage is parallel to the slope. If a plane slip had developed at a depth of 5 m below the surface, determine the factor of safety. Take $\gamma_w = 10\ kN/m^3$.*

Solution. From Eq. 18.17,

$$F_s = \frac{c' + \gamma' H \cos^2 i \tan \phi'}{\gamma_{sat} H \cos i \sin i}$$

$$= \frac{10 + (20 - 10) \times 5 \times \cos^2 10° \tan 25°}{20 \times 5 \times \cos 10° \sin 10°} = \mathbf{1.90}$$

Illustrative Example 18.2. *A vertical cut is made in a clay deposit ($c = 30\ kN/m^2$, $\phi = 0$, $\gamma = 16\ kN/m^3$). Find the maximum height of the cut which can be temporarily supported.*

Solution. From Eq. 18.28,

$$S_n = \frac{c}{F_c\ \gamma H}$$

Taking $F_c = 1.0$,

$$S_n = \frac{30}{1.0 \times 16 \times H} \quad \ldots(a)$$

From stability chart (Fig. 18.13 or Table 18.1), $S_n = 0.261$.

Substituting the value of S_n in Eq. (*a*),

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$$H = \frac{30}{1.0 \times 16 \times 0.261} = \mathbf{7.18\ m}$$

Illustrative Example 18.3. *A cut of depth 10 m is made in a cohesive soil deposit (c = 30 kN/m², ϕ = 0 and γ = 19 kN/m³). There is a hard stratum under the cohesive soil at a depth of 12 m below the original ground surface. If the required factor of safety is 1.50, determine the safe slope.*

Solution. In this case, $D_f H$ = 12 m and H = 10 m. Therefore, D_f = 1.20.

From Eq. 18.28, $$S_n = \frac{c}{F_c \gamma H} = \frac{30}{1.5 \times 19 \times 10} = 0.105$$

From Fig. 18.14, for S_n = 0.105 and D_f = 1.20, we have i = 15°.

Illustrative Example 18.4. *Determine the factor of safety with respect to cohesion for a submerged embankment 25 m high and having a slope of 40°. (c = 40 kN/m², ϕ = 10° and γ_{sat} = 18 kN/m³).*

Solution. From Eq. 18.28, using submerged unit weight,

$$S_n = \frac{40}{F_c \times (18 - 9.81) \times 25} = \frac{0.195}{F_c} \quad \ldots(c)$$

For ϕ = 10° and i = 40°, Table 18.1 gives S_n = 0.097 by interpolation.

Therefore, $$0.097 = \frac{0.195}{F_c} \qquad \text{or } F_c = \mathbf{2.01.}$$

Illustrative Example 18.5. *Determine the factor of safety with respect to cohesion if an embankment 25 m high and having a slope of 40° is subjected to sudden drawdown. (c = 40 kN/m², ϕ = 10°, γ_{sat} = 18 kN/m³).*

Solution. From Eq. 18.30, $$\tan \phi_w = \frac{\gamma'}{\gamma_{sat}} \left(\frac{1}{F_\phi} \tan \phi' \right)$$

Taking F_ϕ = 1.0, $$\tan \phi_w = \frac{(18 - 9.81)}{18} (\tan 10°) \quad \text{or } \phi_w = \mathbf{4.59°}$$

From Table 18.1 (or Fig. 18.13), S_n = 0.13 for φ_m = 4.59° and i = 40°.

From Eq. 18.28, using saturated unit weight,

$$0.130 = \frac{40}{F_c \times 18 \times 25}$$

or $$F_c = 0.684 < 1.00$$

The slope is not safe.

Illustrative Example 18.6. *Determine the factor of safety with respect to shear strength of a slope 10 m high and having an inclination of 40° of a soil with c = 30 kN/m², ϕ = 10° and γ = 19 kN/m³.*

Solution. The factor of safety is obtained by trial and error. Let us assume F_φ = 1.30 for the first trial.

From Eq. 18.4, $$\tan \phi_m = \frac{\tan \varphi}{F\phi} = \frac{\tan 10°}{1.30}$$

or $$\phi_m = 7.72°$$

From Table 18.1, for ϕ_m = 7.72° and i = 40°,

$$S_n \sim 0.115$$

From Eq. 18.28, $$0.115 = \frac{c}{F_c \gamma H}$$

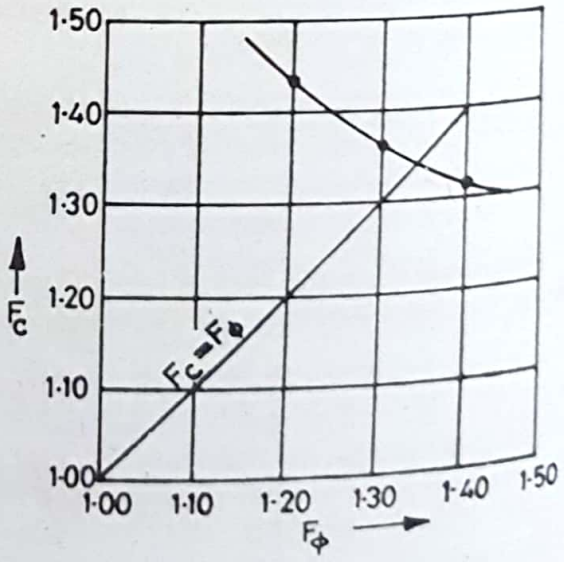

Fig. E-18..

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